

# The Convergence Scheme on Network Utility Maximization in Wireless Multicast Networks

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## ABSTRACT

With the ever-increasing wireless data application recently, considerable efforts have been focused on the design of distributed explicit rate scheme based on Network Utility Maximization (NUM) or wireless multi-hop mesh networks. This paper describes a novel wireless multi-hop multicast flow control scheme for wireless mesh networks via 802.11, which is based on the distributed self-turning Optimal Proportional plus Second-order Differential (OPSD) controller. The control scheme, which is located at the sources in the wireless multicast networks, can ensure short convergence time by regulating the transmission rate. We further analyze the theoretical aspects of the proposed algorithm. Simulation results demonstrate the efficiency of the proposed scheme in terms of fast response time, low packet loss and error ration.

Keywords: multicast, utility maximization, distributed networks, 802.11.

## 1. Introduction

Wireless multicast mesh networks are a promising structure, which can support high quality television broadcasts, ADSL-based broadband Internet connections and so on. However, regular wireless access points (APs) cannot satisfy transmission quality requirements of multimedia applications. Wireless video transmission via multi-hop links is fragile against background data traffic, e.g., web browsing, and the transmission may vary according to the number of the user and the limitation of links. Since 802.11-based multi-hop wireless networks implement Distributed Coordination Function (DCF) for medium access, an end-to-end route may contain several relay nodes contending for the channel resulting degradation of video quality due to increased end-to-end delay and packet probability. All of these problems have prevented the implementation of indoor cabling by 802.11 links until now.

Our proposal approach aims to improve the multimedia experience at home by providing high quality video streaming via multicast wireless mesh technology by fast convergence rate. The self-

tuning multicast rate control helps to establish and maintain high quality links between multi-hops wireless nodes. We overcome the delay and optimal problems by combing wireless mesh networks with dynamic adaptable scheme. We can explain this problem in detail.

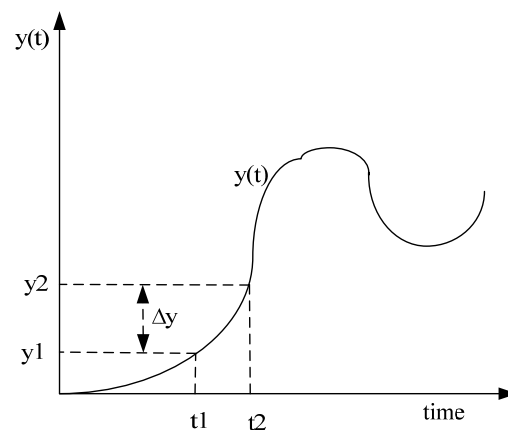


Figure1. Illustration of algorithm motivation.

To explain the delay of rate in multi-hop network, we use Figure 1 to show the problem. In the illustration of Figure 1, we define  $y_1$  be the desired input rate at time  $t_1$ . So, after some time, maybe at time  $t_2$ , the input rate gradually becomes  $y_1$ . However, at time  $t_2$ , the desired input rate should be  $y_2$ . We could find that there is an offset rate between time  $t_2$  and time  $t_1$  which degrades the network performance greatly. This problem is severe in wireless multi-hop networks due to long round-trip. Failure to take this component into consideration may result in oscillation of buffer occupancy and degrade the network performance.

## 2. Related works

In prior work, 802.11 based wireless mesh networks (WMNs) have been mainly investigated for extending wireless transmission throughput [1]. Although WMNs improve the quality of the one point to many points transmission in the networks, contending is also increased in the wireless channel due to 802.11's inherent access mechanism. To overcome increased number of packet losses and delay jitter, both of which are especially harmful for video, the authors of [2] presented enhanced distributed channel access 802.11 (EDCA) to provide quality of service (QoS) via different priority traffic classes with different contention window size and arbitration inter-frame space (AIFs) values. In [3], many schemes are presented to improve the video quality over wireless transmission by designing smart prioritization together with mesh networking. However, beside of signal collapse, the multi-hop wireless links can cause congestion or even congestion collapse if adequate flow control is not provided. Flow control thus plays an important role in the traffic management of multicast multi-hop communications. Without an adequate flow control scheme being implemented, the bottleneck link might cause its buffer to overflow, or cause excessive queuing delay or even deadlock [4]. Then, intensive research has been done on wireless mesh rate control [5], [6], [7]. To overcome multi-hop delay constraints, Hou [8] investigated maximizing throughputs for QoS requirements for distributed wireless mesh networks. Huang et al. [9] addressed the problem of optimal transmission rate using random delay threshold policies. To

resolve the deployment issues of multicast multi-hop distributed wireless network, lots of papers have done deep research [10], [11], [12].

## 3. The OPSD (Optimal Proportional plus Second-order Differential) control scheme

### 3.1 The System Model and Notations

In a wireless multicast network, many multicast sessions are implemented along a multicast tree. The multicast source is the root of the tree. From the source, the multicast traffic flows propagate toward many multicast destinations. In general, a multicast tree is constructed at the beginning of network and it may be changed dynamically according to network topology change, members departing or relay node's failure.

Every wireless router, which located at the branch point in the multicast tree, implements algorithm control function. The branch point is important for multimedia transmission for one time sending data requirement but spacious coverage ability. Due to the popular multimedia transcoding techniques, such as MPEG-4 video standard, many downward nodes can accept video files freely by using fine-grained multimedia techniques [13].

A rate controller is located at the root of every router node. The outgoing links of a router is towards its downstream routers. The basic idea of flow control is as follows: Initially, the source sends data including a forward control packet (FCP) to the destinations along the route. At each router node, it computes an expected incoming data rate according to its local buffer occupancy, and construes a backward control packet (BCP) by filling this expected rate in the BCP.

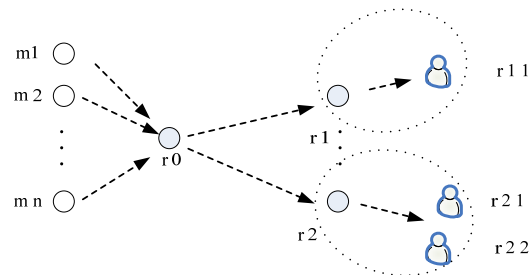


Figure 2. A multicast model in which every wireless router with downstream routers or end users. To explain our distributed control scheme in detail, as shown in Figure 2, we assume there are  $m$

multicast sources  $(m_1, m_2, \dots, m_{|M|})$  in the network. A multicast session may have many end users. For example,  $(m, n)$  means the  $n$ th user belonging to a multicast session  $m$ .

In our algorithms, the aim is to amplify the utility of wireless bandwidth to meet user's demands to the greatest degree. So, we define some notations as below.

Define  $x_{(m,n)}$  as the received rate at the  $n$ 's end user. And define  $x_{(m,\cdot)}$  means the supreme received rate among multicast session  $m$ .

The dynamic multicast optimal problem on NUM can be formulated as:

P1:

$$\max \sum_{m=1}^{|M|} \left( \sum_{n=1}^{|N|} U(x_{(m,n)}) \right) \quad (1)$$

$$\sum_{m=1}^{|M|} x_{(m,\cdot)} \Big|_{m \in f} \leq B_f^{\max} \quad (2)$$

$$B_f^{\max} = W_q \frac{1}{T} \log(1 + K\gamma_f) \quad (3)$$

$$\forall set \quad \nabla = B_l^{\max} - \sum_{m=1}^{|M|} x_{(m,\cdot)}(t) \Big|_{m \in l} \quad (4)$$

$$\begin{aligned} \nabla'(k+1) &= \nabla(k+1) - \nabla(k) \\ &= \sum_{m=1}^{|M|} [x_{(m,\cdot)}(t) - x_{(m,\cdot)}(t+1)] \Big|_{m \in l} \end{aligned} \quad (5)$$

$$\begin{aligned} \nabla''(k+1) &= \nabla(k+1) + \nabla(k-1) - 2\nabla(k) \\ &= \sum_{m=1}^{|M|} [2x_{(m,\cdot)}(t) - x_{(m,\cdot)}(t+1) - x_{(m,\cdot)}(t-1)] \Big|_{m \in l} \end{aligned} \quad (6)$$

So, the Primal problem of multicast network utility maximization (NUM) can be solved by the Lagrangian function as below.

$$\begin{aligned} L(t+1)_{(x, \nabla, \nabla', \nabla'')} &= \sum_{m=1}^{|M|} \left( \sum_{n=1}^{|N|} U(x_{(m,n)}) \right) + \sum_l \rho_l \left[ \sum_{m=1}^{|M|} [B_l^{\max} - x_{(m,\cdot)}(t+1)] \right] \\ &+ \sum_l \rho'_l \left[ \sum_{m=1}^{|M|} [x_{(m,\cdot)}(t) - x_{(m,\cdot)}(t+1)] \right] \\ &+ \sum_l \rho''_l \left[ \sum_{m=1}^{|M|} [2x_{(m,\cdot)}(t) - x_{(m,\cdot)}(t+1) - x_{(m,\cdot)}(t-1)] \right] \end{aligned}$$

The Primal problem can be changed into the Dual problem D1

D1:

$$\min_{\rho, \rho', \rho'' \geq 0} D(t+1)_{(x)} \quad (7)$$

According to the Dual problem, the multicast source node  $m$  updates in according to the rules below:

$$x_m(t+1) = x_m(t) + \varepsilon_m(t) \Gamma_m(t) \quad (8)$$

In Eq.8,  $\Gamma_m(t)$  is an estimated system value including noises, which is calculated by:

$$\Gamma_m(t) = \Gamma_m^\circ(t) + o_m^b(t) + o_m^m(t) \quad (9)$$

where  $\Gamma_m^\circ(t)$  is the practical value from a sub-gradient method in Eq. (10).

$$\Gamma_m^\circ(t) = \partial U / \partial x_m - \sum_{l \in m} (\rho(t) + \rho'(t) + \rho''(t)) \quad (10)$$

In Eq.10, the temporal variables  $\rho(t)$ ,  $\rho'(t)$ ,  $\rho''(t)$  can be calculated in each wireless node at the branch of virtual multicast tree as :

$$\rho_l(t+1) = \rho_l(t) - \varepsilon_l(t) \Gamma_l(t) \Big|_0^\infty,$$

$$\rho'_l(t+1) = \rho'_l(t) - \varepsilon'_l(t) \Gamma'_l(t) \Big|_0^\infty$$

$$\text{and } \rho''_l(t+1) = \rho''_l(t) - \varepsilon''_l(t) \Gamma''_l(t) \Big|_0^\infty.$$

Moreover, the  $\Gamma_l(t)$ ,  $\Gamma_l'(t)$ ,  $\Gamma_l''(t)$  can be obtained by

$$\begin{aligned} \Gamma_l(t) &= \Gamma_l^\circ(t) + o_l^b(t) + o_l^m(t), \\ \Gamma_l'(t) &= \Gamma_l'^\circ(t) + o_l'^b(t) + o_l'^m(t) \\ \text{and } \Gamma_l''(t) &= \Gamma_l''^\circ(t) + o_l''^b(t) + o_l''^m(t). \end{aligned}$$

For clear understanding, we introduce the notations as below.

$x_m(t)$	The transmitting rate of multicast session $m$ at time $t$
$\Gamma_m(t)$	The estimated value of $\Gamma_m^\circ(t)$
$\Gamma_m^\circ(t)$	The practical value from a sub-gradient method
$\Gamma_l(t)$	The estimated value of $\Gamma_l^\circ(t)$
$\Gamma_l^\circ(t)$	The practical value from a sub-gradient method
$\Gamma_l'(t)$	The estimated value of $\Gamma_l'^\circ(t)$
$\Gamma_l'^\circ(t)$	The practical value from a sub-gradient method
$\Gamma_l''(t)$	The estimated value of $\Gamma_l''^\circ(t)$
$\Gamma_l''^\circ(t)$	The practical value from a sub-gradient method
$o_m^b(t)$	The biased estimation error of $\Gamma_m^\circ(t)$
$o_m^m(t)$	The martingale difference noise of $\Gamma_m^\circ(t)$
$o_l^b(t)$	The biased estimation error of $\Gamma_l^\circ(t)$
$o_l^m(t)$	The martingale difference noise of $\Gamma_l^\circ(t)$
$o_l'^b(t)$	The biased estimation error of $\Gamma_l'^\circ(t)$
$o_l'^m(t)$	The martingale difference noise of $\Gamma_l'^\circ(t)$
$o_l''^b(t)$	The biased estimation error of $\Gamma_l''^\circ(t)$
$o_l''^m(t)$	The martingale difference noise of $\Gamma_l''^\circ(t)$

Table 1. Notations in the algorithms.

### 3.2 System optimal analysis

**Theorem 1:** The algorithms of Eqs.1, 2, 3, 4, 5 and 6 converge in distribution to global optimum with probability 1 under the conditions of A1 – A4.

**A1)** The noise terms  $\Gamma_l(t)$ ,  $\Gamma_l'(t)$ ,  $\Gamma_l''(t)$ ,  $\Gamma_m(t)$  are independent on iterations.

**A2)**  $\varepsilon(t) > 0$ ,  $\varepsilon(t) \rightarrow 0$ ,  $\sum_{n=0}^{\infty} \varepsilon^2(t) < \infty$  ( $\forall \varepsilon_l(t)$ ,  $\varepsilon_l'(t)$ ,  $\varepsilon_l''(t)$ ,  $\varepsilon_m(t)$ ).

**A3)** The biased errors  $o_l^b(t)$ ,  $o_l'^b(t)$ ,  $o_l''^b(t)$ ,  $o_m^b(t)$  have nonnegative constant bound.

**A4)** The martingale difference noises have bound:  
 $\sup E[o_l^m(t)^2] < \infty$  &  $\sup E[o_l'^m(t)^2] < \infty$   
 &  $\sup E[o_l''^m(t)^2] < \infty$  &  $\sup E[o_m^m(t)^2] < \infty$ .

**Proof:** The convergence proof is a special case of work [14]. We give a sketch of proof as follows.

Let  $(x^*, \rho^*, \rho'^*, \rho''^*)$  be the point where  $x^*$  is the unique optimum solution to the primal problem.

Define the Lyapunov function  $V(x, \rho, \rho', \rho'')$  as below:

$$\begin{aligned} V(x, \rho, \rho', \rho'') &= (x - x^*)^2 + \min(\rho - \rho^*)^2 + \min(\rho' - \rho'^*)^2 \\ &\quad + \min(\rho'' - \rho''^*)^2 \end{aligned}$$

And define the set

$$A_\Omega \equiv \{(x, \rho, \rho', \rho'') : V(x, \rho, \rho', \rho'') \leq \Omega\}$$

There can be many  $(x(t), \rho(t), \rho'(t), \rho''(t))$  within any definite  $\Omega > 0$ .

Consider  $\Gamma_l(t)$  for instance. Since  $\Gamma_l(t)$  has the boundary, combining **A1**, it yields  $\varepsilon_l(t)\Gamma_l(t) \rightarrow 0$ .

Considering **A2**, we get  $\varepsilon_l(t)o_l^b(t) \rightarrow 0$ . For any small positive  $\sigma$ , use Chebyshev's Inequality, one has

$$P(|\varepsilon_l(t)o_l^b(t)| > \sigma) \leq \varepsilon_l(t)^2 E[o_l^b(t)^2] / \sigma^2 \quad (11)$$

Using Borel-Cantelli Lemma and **A4**, we conclude that  $\varepsilon_l(t)o_l^b(t) > \sigma$  is finite with probability 1 [15].

Then, using the Martingale Inequality[16] and **A3**, we can proof that  $\Omega$  can be selected unlimited small, so  $\{x(t), \lambda(t), \mu_q(t), \mu_\Delta(t), t \rightarrow \infty\}$  converges with probability 1 to the optimal point.

Theorem 1 proves the existence of stability and convergence of distributed algorithms of NUM under feedback noises and control delay in wireless mesh networks.

#### 4. Performance

To evaluate the performance of the proposed multicast rate control method, we pay more attention to the related important parameters: sending rates of source nodes, convergence time and network utilization in our distributed algorithms.

We assume Signal to Interference Ratio (SIR) for the  $i^{th}$  logical link is defined as

$$\gamma_i(P) = \frac{P_i G_{ii}}{\sum_{j \neq i}^N P_j G_{ij} + N_i} \quad (12)$$

where  $G_{ij}$  is the path loss from the transmitter on logical link  $j$  to the receiver on logical link  $i$ , taking into account propagation loss and normalization factors, and  $G_{ii}$  is the path gain for the intended transmission on logical link  $i$ , taking into account propagation loss and other factors such as spreading gain and the effect of beam forming. For a large class of modulation, the attainable data rate can be written as Eq.12. Since systems are interference limited where  $N_i$  is much smaller than the total interference,  $\gamma_i$  is approximated as

$$\gamma_i = \frac{P_i G_{ii}}{\sum_{j \neq i}^N P_j G_{ij}}$$

Assume the wireless mesh network uses  $U(x_{(m,n)}) = \log x_{(m,n)}$  to achieve fairness among flows. Table 2 presents the parameters of the simulations.

$\varepsilon_l$	$\varepsilon_l = 0.45\varepsilon'_l$
$\varepsilon'_l$	$1/n$ ( $n$ is the iteration)
$\varepsilon''_l$	$1/2n$ ( $n$ is the iteration)
$\varepsilon_m$	$1/n$ ( $n$ is the iteration)
$K$	1
$W_q$	10MHz
$N_i$	3mW
$P_i$	100Mw
$P_j$	100Mw
Delay	20ms ~ 50ms
Topology region	$500 \times 500 m^2$
Node number	20

Table 2. Parameters of the simulations.

There are 20 nodes in a large area in which two multicast sessions are deployed. The results are shown in Fig.3~6.

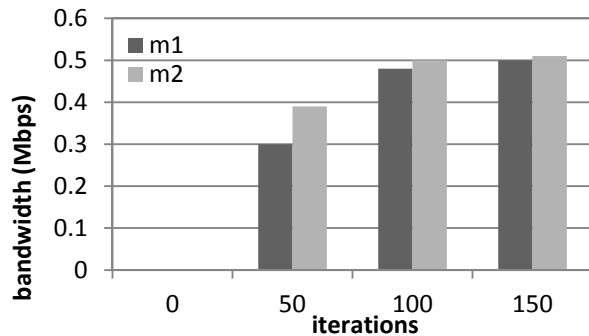


Figure 3. Bandwidths of m1 and m2 at R0 (Iterations between 0 to 150).

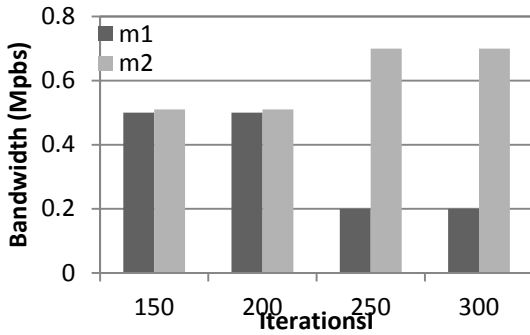


Figure 4. Bandwidths of m1 and m2 at R0 (Iterations between 150 to 300).

Figure 3 gives the bandwidth change of m1 and m2 at R0 when the iterations from 0 to 150. Figure 4 gives the bandwidth change of m1 and m2 at the same location when the iterations from 150 to 300. We find the bandwidth curves of m1 and m2 reach the steady state after 100 iterations time.

To test our distributed algorithms, we delete one end user from multicast session m1 at 150 iterations. We can find that the bandwidth of m1 decreases from 0.5Mbps to 0.2Mbps. However, in the same time, the bandwidth of m1 increases slightly.

Figure 5 gives the utility change of m1 and m2 when the iterations from 0 to 150. Figure 6 gives the utility change at the same location when the iterations from 150 to 300.

We can see the utility of m1 and m1 are stable after 120 iterations. However, according to user departing, the utility of m1 decreases from 1.2 to 0.8Mbps. However, in the same time, the utility of m2 increases slightly.

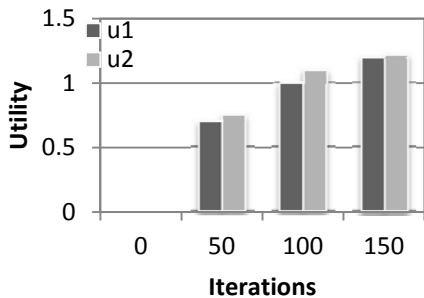


Figure 5. Utility of m1 and m2 (Iterations between 0 to 150).

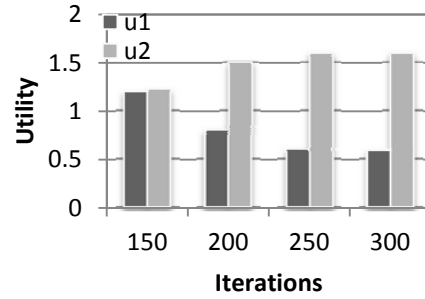


Figure 6. Utility of m1 and m2 (Iterations between 150 to 300).

	Scenario 1 (iterations)	Scenario 2 (iterations)
Before	250	300
After	120	140

Table 3. Convergence time after adopting OPSD algorithms.

To test the convergence time of the optimal distributed multicast algorithms, two scenarios are presented. In the first scenario, there are 4 end users who receive 2 multicast sessions respectively. In the second scenario, there are 6 end users who received 2 multicast sessions respectively. That is to say, two users are added to be the multicast session receivers. Table 3 compared the difference of convergence time after applying the OPSD with the time with before using it. From table3, we can find the convergence time is shorter after implementing the OPSD algorithms. Thus, our distributed multicast rate control method ensures good system performance.

### 5. Conclusion

In this paper, we try to resolve the problem of slow transient response due to the long feedback time in wireless multicast networks. Delayed congestion feedback can cause excessive queue buildup and packet loss at bottleneck links especially in multicast. We proposed a novel Optimal Proportional plus Second-order Differential (OPSD) control which has capability to diminish oscillations in distributed wireless mesh networks via 802.11. The merit of our distributed algorithms is that they can achieve the optimums quickly. As evident from the analyses and simulation results, the proposed control scheme can improve the network performance.

Our further research along this line of study would investigate how to choose control parameters according to delay time in distributed wireless multicast mesh networks.

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### References

- [1] G.R. Hiertz et al., "Principles of IEEE 802.11s," in Proceeding of 16th International Conference on Computer Communications and Networks, Turtle Bay Resort, Honolulu, Hawaii, USA, 2007. pp. 1002-1007.
- [2] "IEEE Standard for Information technology—Telecommunications and information exchange between systems—Local and metropolitan areas networks—Specific requirements Part11: Wireless LAN Medium Access Control (MAC) Quality of Service Enhancements," IEEE Std. 802.11e-2005 (Amendment to IEEE Std 802.11, 1999 Edition), 2005, pp.1-189.
- [3] F. Birlik et al., "IPTV Home Networking via 802.11 Wireless Mesh Networks: An Implementation Experience," IEEE Transaction on Consumer Electronics, vol. 55, no. 3, pp. 326-337, August 2009.
- [4] N. Xiong et al., "Distributed Explicit Rate Schemes in Multi-Input-Multi-Output Networks Systems," IEEE Transaction on Systems, Man, and Cybernetics-Part C: Applications and Reviews, vol. 40, no. 4, pp.1254-1266, July 2010.
- [5] Y. Song et al., "Stochastic Traffic Engineering in Multihop Cognitive Wireless Mesh Networks," IEEE Transaction on Mobile Computing, vol. 9, no. 3, pp. 124-132, 2010.
- [6] J. Guo and A. Liu, "A New Distributed Topology Control Algorithm Based on Optimization of Delay and Energy in Wireless Networks" , Journal of Parallel and Distributed Computing, vol.72, no.8, pp. 1032-1044, 2012.
- [7] V. Spyridon and Y. Gregory, "Shortest Route Mobility Assisted Packet Delivery with Soft Maximum Delay Guarantees in Mobile ad hoc Networks," Journal of AD HOC Networks, vol.10, no.6, pp.886-900, 2012.
- [8] I. Hou and P. R. Kumar, "Utility Maximization for Delay Constrained QoS in Wireless," in INFOCOM, San Diego, CA, USA, 2010, pp.1-9.
- [9] J. Huang and V. Krishnamurthy, "Transmission Control in Cognitive Radio as a Markovian Dynamic Game: Structural Result on Randomized Threshold Policies," IEEE Transactions on Communications, vol. 58, no.1, 2010, pp.301-310.
- [10] J. Zhang et al., "The Impact of Stochastic Noisy Feedback on Distributed Network Utility Maximization," IEEE Transactions on Information Theory, vol. 54, no. 2, 2008, pp.231-244 .
- [11] B. Park et al., "QoS-Driven Wireless Broadband Home Networking Based on Multihop Wireless Mesh Networks," IEEE Transactions on Consumer Electronics, vol.52, no.4, pp.1224-1238, 2006.
- [12] A. Asaduzzaman and H.Y.Kong, "Muti-Relay Cooperative Diversity Protocol with Improved Spectral Efficiency," Journal of Communications and Networks, vol. 13, no.3, 2011, pp. 679-688.
- [13] D. Rubenstein et al., "The Impact of Multicast Layering on Network Fairness," IEEE/ACM Transaction on Networking, vol.10, no.2, pp.169-182, 2002.
- [14] J. Zhang et al., "The Impact of Stochastic Noisy Feedback on Distributed Network Utility Maximization," IEEE Transactions on Information Theory, vol. 54, no. 2, 2008, pp.1215-1226.
- [15] P. Billingsley, "Probability and Measure," the third edition. New York: Wiley, 1995, pp.106-189.
- [16] Y. Shi and T. Hou, "A Distributed Optimization Algorithm for Multi-Hop Junshan Cognitive Radio Networks", in INFOCOM, Phoenix, AZ, 2008, pp. 1292-1300.